QUALITATIVE RESULTS FOR DISTRIBUTED SYSTEMS WITH DISCRETE DAMPING AND STIFFNESS WITH APPLICATION TO CONTROL, INTERIM SCIENTIFIC REPORT,

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Research Objectives

The research carried out under this grant [1] considered the vibration and control of a linear non-conservative dynamic systems described by partial differential equations of the form

$$u_{tt}(x,t) + L_1 u_t(x,t) + L_2 u(x,t) = f(x,t) \text{ on } \Omega \qquad (1)$$

$$Bu(x,t) = 0 \text{ on } \partial\Omega$$

where L_1 and L_2 are time invariant partial differential operators defined on a Hilbert Space, H_D, consisting of all appropriately smooth functions satisfying the boundary conditions, Ω is a bounded open region in R^n, $n = 1,2,3$, with boundary $\partial\Omega$, t denotes the time, and $u(x,t)$ is the deflection at the position x in Ω. The subscript t denotes differentiation with respect to the time, t. The boundary conditions are represented by the time invariant partial differential operator B evaluated along the boundary $\partial\Omega$. Further assumptions are made to insure the existence of a solution of (1) of the form of a modal series.

The specific research objectives as listed in (1) are:

(1) Extend the theory developed in [2-4] to include those problems which have non self-adjoint operators L_1 and L_2 but rather for which there exists an operator P defined on H such that the operators PL_i are self adjoint.

(2) Use test functions to describe point actuators combined with the oscillation theorem of [2] to develop tighter bounds on the residual modes of the control problem and to extend these bounds to more general non conservative systems.

The next section describes the extent to which these objective have been met.

Accomplishments

Objective (1) has been partially met by the following theorem, presented at the 1983 SIAM Winter Meeting in November, which can be stated for (1) for the special case of $L_1 = 0$ and L_2 non-self adjoint.

Theorem 1: If there exists a self adjoint, positive definite operator P with compact resolvent such that the product operator PL_2 is a non trival self adjoint operator with compact resolvent, then there exists a complete set of eigenfunctions, $\{\phi_n(X)\}$, such that

$$u(x,t) = \sum_{n=1}^{\infty} A_n(t)\phi n(X)$$

is a solution of (1). Furthermore, the stability results and convergence results listed in [3] hold for under this case.

This result shows that modal control methods can be used in some non self adjoint cases. The physical problem known as a Phlüeger's column offers a non trival example of such a distributed parameter system.

Objective (11) has been more difficult to achieve and to date has only yielded results primarily for finite dimensional approximations. Of course the spirit of this work is to produce results based soley on the distributed formulation, so that this work is viewed as preliminary. Results obtained so far indicate that tight bounds can be found for the transcient response of an approximate system. The bounds are stated in terms of the relative values of the physical parameters of the system and can be shown to be better than existing theories when the eigenvalues of the stiffness operator are spread out and when the finite model has a large number of modes. This is encouraging for the fully distributed case.

In the special case that the stiffness and damping operators commute, simple, gross bounds can be found for the residual modes in the control problem and are given in [5], a copy of which is appended.

It has also been shown that the condition previously discovered by Gibson [3] can be derived from the underdamping condition given in [4]. This condition insures that optimal control laws derived based on finite dimensional models converge to optimal control for the full distributed system and that the response is uniformly exponentially stable. The result derived under this grant yields the physical interpretation that such systems are underdamped (but damped) systems [6].

Publication List

The following lists those publications appearing in, submitted to, or in preparation for, referred journals and proceedings representing work performed under this grant and acknowledged as such:

Inman, D.J. "Modal Decoupling Conditions for Distributed Control of Flexible Structures", AIAA Journal of Guidance Control and Dynamics, July-August 1984.

Nicholson, D.W. and Inman, D.J., "Stable Response of Damped Linear Systems", The Shock and Vibration Digest, Vol. 15, No. 11, 1983, pp. 19-25.

DeCaro, S.M. and Inman, D.J., "Eigenvalue Placement by Constrained Optimization", to appear in the Proceedings of the JPL Workshop on Identification and Control of Flexible Space Structures.

Inman, D.J., "Finite Control of Undamped Distributed Parameter Systems", to appear in the Proceedings of the JPL Workshop on Identification and Control of Flexible Space Structures.

Inman, D.J., "Critical Damping in Complex Structures", to appear in the Proceedings of the Air Force Vibration Damping Workshop, August 1984.

Ng, C.K. and Inman, D.J., "Active Control of Decoupled Underdamped Systems", Proceedings of the 25th Structures Dynamics and Materials Conference, May 1984, pp. 573-577.

Inman, D.J. and Hsieh, C.I., "Controllability of Non-Self Adjoint Flexible Systems", ASME Paper #83-WA/DSC-16, Winter Annual Meeting, November 1983.

Publication List (continued)

Inman, D.J., "On a Class of Non-Self Adjoint Control Problems", in preparation for SIAM Journal of Control and Optimization.

Inman, D.J., "Underdamped Combined Dynamical Systems", in preparation for Quarterly of Applied Mathematics.

Inman, D.J., "On Compact Feedback and Underdamped Distributed Parameter Systems", in preparation for SIAM Journal of Control and Optimization.

Associated Professional Personnel

The following is a list of graduate students, degree (degree sought), thesis title, and date awarded (expected) for those students advised by the principal investigator during this grant period (July 1, 1983 to June 30, 1984) who worked on research related to the grant. Those students who received some support are indicated by an asterisk. (all are U.S. citizens, except Ahmadian)

Musiol, Irene, "Bounds on the Forced Response of Distributed Parameter Systems", M.S. Project, March 1984.

Schultz, Mark, "Study of Damper Placement for Beam Vibration Control", M.S. Project, March 1984.

DeCaro, Sandra*, "Eigenvalue Assignment for Non-Conservative Dynamic Systems", M.S. Thesis, March 1984.

Ahmadian, Mehdi*, "Dynamics and Control of Asymmetric Systems", Ph.D. Dissertation, June 1984.

Yae, Kwang*, "Estimates of Decay Rates for Underdamped Distributed Parameter Systems", Ph.D. exoected, 1986.

Zimmerman, David, "Digital Control of the MASA-UVA Proof Mass Actuator", M.S. Thesis, April 1984,

Zimmerman, David, "Reliability Problems in Large Flexible Space Structures", Ph.D. expected 1986.

Associated Professional Personnel (continued)

Ghamine, George, "Numerical Solutions of Asymmetric Systems", M.S. Project, April 1984.

Cudney, Harley, "Digital Control of a Beam with Joint", M.S. Thesis expected 1984-85.

Ebbing, David, "Damping in Composite Materials", M.S. Thesis expected 1984-85.

Hendrickson, William, "Normal Modes in Composite Materials", M.S. Thesis expected 1984-85.

Duke, Patricia, "Dynamic Stiffness of Control Systems", M.S. Thesis expected 1985.

Interactions

The following is a list of presentations made or to be made by the principal investigator at conferences, meetings and seminars representing work performed under the grant during the period from July 1, 1983 to June 30, 1984.

"Control and Vibrations of Non-Self Adjoint Distributed Parameter Systems", Seminar Series Department of Electrical Engineering, University of Rochester, October 1983.

"Modal Control of a Class of Non-Self Adjoint Systems", SIAM 1983 Fall Meeting, Norfolk, Virginia, November 1983.

"Controllability of Non-Self Adjoint Flexible Systems", ASME 1983 Winter Annual Meeting, Boston, Massachusetts, November 1983.

"Vibration and Control of Large Flexible Space Structures", Seminar Series Department of Mechanical Engineering, Ohio State University, January 1984.

"Vibration Control of Space Structures", Western New York High Technology Lecture Series, Buffalo, New York, January 1984.

"Modal Analysis in Non-Conservative Dynamic Systems", 2nd International Conference on Modal Analysis, Orlando, Florida, 1984

"Critical Damping in Complex Structure", Air Force Vibration Damping Workshop, Long Beach, California, February 1984.

Interactions (continued)

"Active Control of Decoupled Underdamped Systems", 25th Structures Dynamics and Materials Conference, Palm Springs, California, May 1984.

"Microprocessor Control of the NASA-UVA Proof Mass Actuator", AIAA Dynamics Specialist Conference, Palm Springs, California, May 1984.

"Frequency Domain Analysis of a Plate with Discrete Elements Attached", Dynamics Specialist Conference, Palm Springs, California, May 1984.

"Eigenvalue Assignment by Constrained Optomization", JPL Workshop on Identification and Control of Flexible Space Structures, San Diego, California, June 1984.

"Finite Control in Underdamped Distributed Parameter Systems", JPL Workshop on Identification and Control of Flexible Space Structures, San Diego, California, June 1984.

"Research in Distributed Parameter Control Theory", AFOSR Forum on Large Space Structures, McLean, Virginia, June 1984.

"Modes and Critical Damping in Asymmetric Linear Dynamic Systems", XVI ITACM, Lyngby, Denmark, to be presented August 1984.

"Decay Rates for Linear Dynamical Systems", Invited, 21st Society of Engineering Science Meeting, Blackburg, Virginia, to be presented October 1984.

References

[1] Inman, D.J., "Qualitative Results for Distributed Systems with Discrete Damping and Stiffness with Application to Control", AFOSR Grant #820242, January 1984 (and December 1983).

[2] Inman, D.J. and Andry, A.N. Jr., "The Nature of the Temporal Solutions of Damped Linear Distributed Parameter Systems with Normal Modes", Journal of Applied Mechanics, Vol. 49, 1982, pp. 867-870.

[3] Gibson, J.S., "An Analysis of Optimal Modal Regulation: Convergence and Stability", SIAM Journal of Control and Optimization, Vol. 19, 1981, pp. 686-706.

References (continued)

[4] Inman, D.J., "Oscillatory Damped Distributed Parameter System", Mechanics Research Communications, Vol. 9, No. 2, 1982, pp. 101-107.

[5] Ng, C.K. and Inman, D.J., "Active Control of Decoupled Underdamped Systems", Proceedings of the AIAA 25th SIAM Conference Part II, 1984, pp. 192-200.

[6] Inman, D.J., "Finite Control in Underdamped Distributed Parameter Systems", JPL Workshop on Identification and Control of Flexible Space Structures Proceedings, 1984.